**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2019**

**Calculator-free**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 1 of term 4, 2019**

**Question 1 (4 marks)**

|  |
| --- |
| Solution |
|          Equation of the curve that passes through the point  is   |
| Mathematical behaviours | Marks |
| * anti-differentiates using
* anti-differentiates using a factor of one-third
* substitutes the coordinates  into the anti-derivative function to determine the constant of integration correctly
* states the equation of the curve containing  correctly
 | 1111 |

**Question 2(a) (3 marks)**

|  |
| --- |
| Solution |
|  Since then with   we obtain   |
| Mathematical behaviours | Marks |
| * identifies the correct substitution
* integrates correctly
* evaluates the indefinite integral at the end points
 | 111 |

**Question 2(b) (3 marks)**

|  |
| --- |
| Solution |
|  Since it follows that   |
| Mathematical behaviours | Marks |
| * simplifies the integral to requiring the anti-derivative of
* integrates correctly
* evaluates the indefinite integral at the end points
 | 111 |

**Question 2(c) (3 marks)**

|  |
| --- |
|  Solution |
|  If we put  then we find that    |
| Mathematical behaviours | Marks |
| * calculates  correctly
* substitutes into integral changing the limits appropriately
* integrates the expression correctly
 | 111 |

**Question 2 (d) (3 marks)**

|  |
| --- |
| Solution |
|  If we write  then we conclude that  and  whence  and .Hence    |
| Mathematical behaviours | Marks |
| * writes the correct form of the partial fractions
* deduces the correct values of the constants and
* evaluates the integral correctly
* no penalty for omitting the arbitrary constant
 | 111 |

**Question 3(a) (4 marks)**

|  |
| --- |
| Solution |
| Assuming $a=2, $the system of equations reduces to$x+0.5y+0.5z=0.5$ $0.5y-4.5z=-0.5$ $0.5y-2.5z=1.5$ (\*)and then to$x+0.5y+0.5z=0.5$ $y-9z=-1$ $2z=2$ (\*\*)So $z=1$Back substitution gives $y=8$ and $x=-4$  |
| Mathematical behaviours | Marks |
| * eliminates first variable (\*)
* eliminates second variable (\*\*)
* back substitutes for second variable
* back substitutes for first variable
 | 1111 |

**Question 3(b) (2 marks)**

|  |
| --- |
| Solution |
| The system of equations reduces to$x+0.5y+0.5z=0.5$ $0.5y-4.5z=-0.5$ $0.5y+(a-4.5)z=1.5$ and then to$x+0.5y+0.5z=0.5$ $y-9z=-1$ $az=2$ (\*)There is no solution if $a=0$ |
| Mathematical behaviours | Marks |
| * eliminates first two variables (\*)
* obtains correct answer
 | 11 |

**Question 4(a)(i) (1 mark)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * verifies the result
 | 1 |

**Question 4(a)(ii) (2 marks)**

|  |
| --- |
| Solution |
|  The equation has solutions so that  or . |
| Mathematical behaviours | Marks |
| * uses quadratic formula identifying the appropriate square root
* deduces the two solutions
 | 11 |

**Question 4(b) (2 marks)**

|  |
| --- |
|  Solution |
| Since  we conclude that  |
| Mathematical behaviours | Marks |
| * multiplies by the complex conjugate
* writes in polar form
 |  1 1 |

**Question 4(c) (2 marks)**

|  |
| --- |
| Solution |
|  As this means that if the complex number is multiplied by the effect is to double the distance of the point in the Argand diagram from the origin. Moreover, the line joining the origin to is rotated anticlockwise through an angle   |
| Mathematical behaviours | Marks |
| * recognises the effect is to double the distance from O
* comments that there is an anticlockwise rotation through
 | 11 |

**Question 5(a) (2 marks)**

|  |
| --- |
| Solution |
| The general equation of a plane is $r.n=c,$ so $2i+j-2k$is a normal to the plane. (\*)Since $\left‖2i+j-2k\right‖=\sqrt{2^{2}+1^{2}+(-2)^{2}}=\sqrt{9}=3,$ $2/3i+1/3j-2/3k$is a unit normal to the plane.(So too is $-2/3i+1/3j-2/3k$.)  |
| Mathematical behaviours | Marks |
| * obtains a normal to the plane (\*)
* divides by the length to get a unit vector
 | 11 |

**Question 5(b) (3 marks)**

|  |
| --- |
| Solution |
| If $A(a,b,c)$ is the point on the plane closest to the origin then $2a+b-2c=18$ and $ai+bj+ck$is normal to the plane. So $ai+bj+ck=t(2i+j-2k)$ for some real number $t.$So $2(2t)+t-2(-2)t=18, $i.e. $9t=18$ and so $t=2$.So the coordinates of $A$ are $(4,2,-4$)  |
| Mathematical behaviours | Marks |
| * recognizes that $OA$ is normal to the plane
* solves for the parameter $t$
* obtains correct answer
 | 111 |

**Question 6(a) (4 marks)**

|  |
| --- |
| Solution |
|     |
| Mathematical behaviours | Marks |
| * indicates asymptote at  correctly
* indicates asymptote at  correctly
* indicates y-intercept at (0,1) and x-intercept at
* correct shape of the curve
 | 1111 |

**Question 6(b) (2 marks)**

|  |
| --- |
|  Solution |
|    and  will intersect at 2 points with positive x-coordinatesi.e.   has 2 non-negative roots if  |
| Mathematical behaviours | Marks |
| * states that
* states that
 | 11 |

**Question 7(a) (2 marks)**

|  |
| --- |
| Solution |
|   or  |
| Mathematical behaviours | Marks |
| * uses tangent ratio to form a correct equation
* expresses  in terms of  correctly
 | 11 |

**Question 7(b) (2 marks)**

|  |
| --- |
|  Solution |
|   |
| Mathematical behaviours | Marks |
| * uses quotient rule to differentiate correctly
* states correct expression in simplest form
 | 11 |

**Question 7(c) (4 marks)**

|  |
| --- |
| Solution |
|  When *x* = 5,  (towards O) and  so that Then    radians/sec  |
| Mathematical behaviours | Marks |
| * identifies
* determines value of correctly when *x* = 5
* identifies correct expression for
* uses chain rule correctly with appropriate substitution to evaluate the value of
 | 1111 |

**Question 8(a) (2 marks)**

|  |
| --- |
| Solution |
|  This is false.A confidence interval may contain none of the underlying population. (For example, if the population consists of an equal number of $0's$ and $1's$, and if the sample is large enough, then a 90% confidence interval will be a small interval of numbers near $0.5$ and will contain neither $0$ nor $1.$   |
| Mathematical behaviours | Marks |
| * gives correct answer
* gives a valid reason
 | 11 |

**Question 8(b) (2 marks)**

|  |
| --- |
| Solution |
|  This is false.The probability that any one of the confidence intervals will contain $μ$ is 0.95. But it is possible that NONE of the twenty confidence intervals will contain $μ.$  |
| Mathematical behaviours | Marks |
| * gives correct answer
* gives a valid reason
 | 11 |

**Question 8(c) (2 marks)**

|  |
| --- |
| Solution |
|  This is true.The width of the confidence interval is proportional to $1/\sqrt{n}$. So to halve the width the sample size must be reduced by a factor of 4  |
| Mathematical behaviours | Marks |
| * gives correct answer
* gives a valid reason
 | 11 |